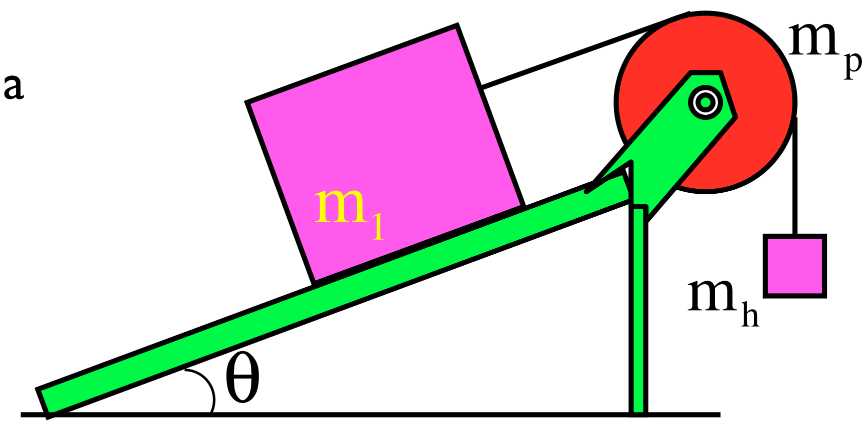
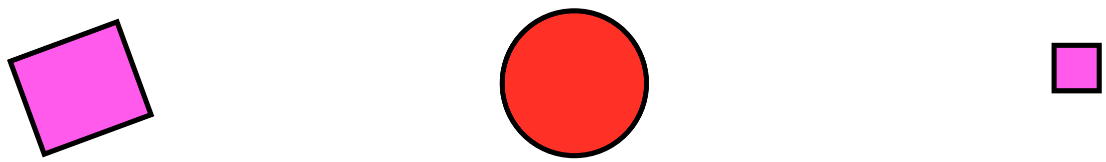
4.) A hanging mass is attached to a string which is threaded over a massive pulley and attached to a second mass sitting on a frictionless incline. Known is:

$$m_1,\ m_h,\ m_p,\ R,\ g,\ \theta,\ \text{and}\ I_{\text{cm of pulley}} = \frac{1}{2} m_p R^2$$

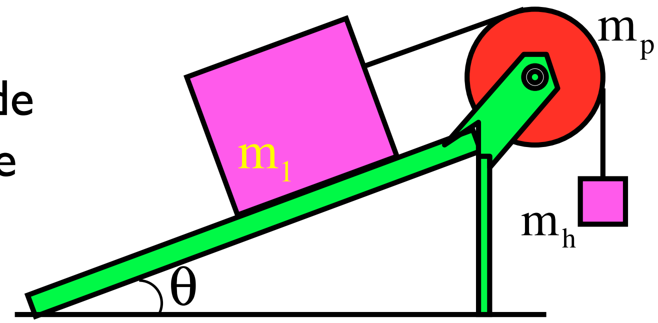


a.) Ignoring the forces acting at the pulley's pin, draw a f.b.d. identifying all the forces acting on both masses and the pulley.



b.) Determine the *moment of inertia* of an axis R/3 units from the center of and perpendicular to an axis through the axle of the pulley.

m_1, m_h, m_p, R, g, θ, and $I_{\text{cm of pulley}} = \frac{1}{2} m_p R^2$

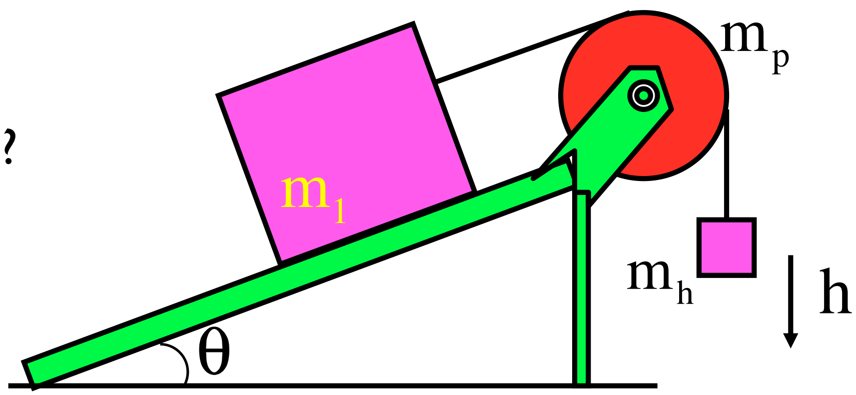


c.) The system begins to accelerate. What is the magnitude of that *acceleration*? (Assume the acceleration is down the incline.)

d.) What is the pulley's *angular acceleration*?

$$m_1, m_h, m_p, R, g, \theta, \text{ and } I_{cm\ of\ pulley} = \frac{1}{2}m_pR^2$$

e.) The hanging mass drops from rest a distance "h." What is its *velocity* magnitude by the end of the drop? (This could happen if the hanging mass was slowing down.)

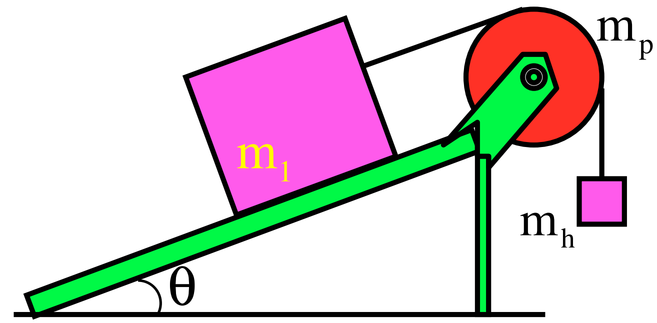


f.) What is the *angular velocity* of the pulley at that point?

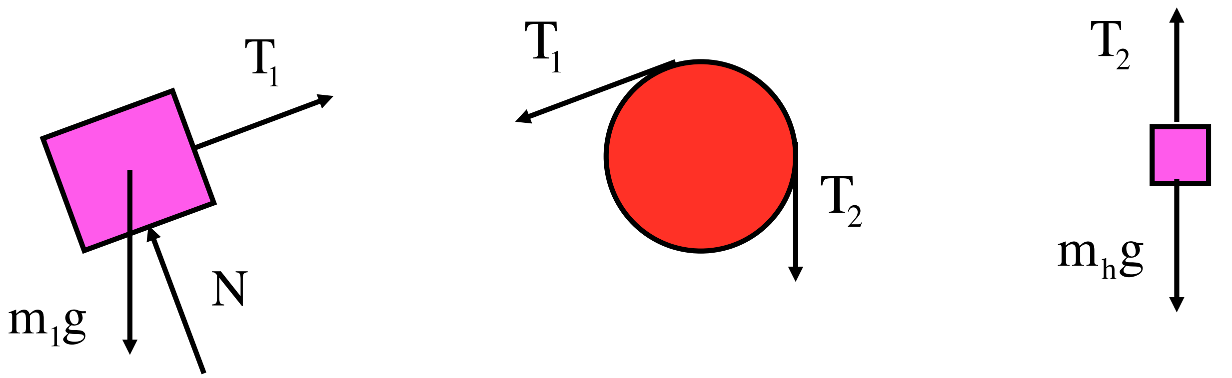
g.) What is the *angular momentum* of the pulley at that point?

4.) A hanging mass is attached to a string which is threaded over a massive pulley and attached to a second mass sitting on a frictionless incline. Known is:

$$m_1,\ m_h,\ m_p,\ R,\ g,\ \theta,\text{ and } I_{\text{cm of pulley}} = \frac{1}{2}m_pR^2$$



a.) Ignoring the forces acting at the pulley's pin, draw a f.b.d. identifying all the forces acting on both masses and the pulley.



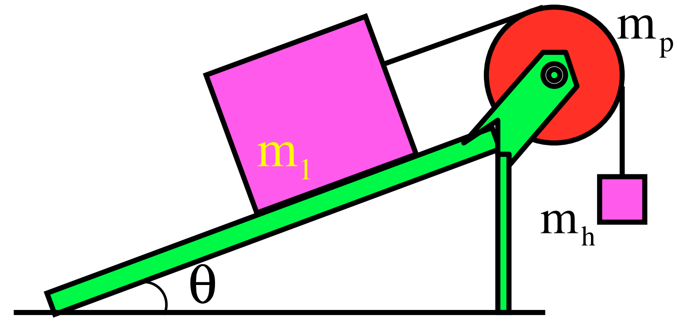
b.) Determine the *moment of inertia* of an axis R/3 units from the center of and perpendicular to an axis through the axle of the pulley.

Using the parallel axis theorem, we get:

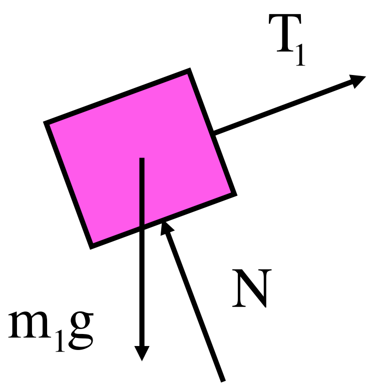
$$\begin{aligned} I_p &= I_{cm} + md^2 \\ &= \frac{1}{2}m_pR^2 + m_p\left(\frac{R}{3}\right)^2 \\ &= \frac{11}{18}m_pR^2 \end{aligned}$$

$m_1,\ m_h,\ m_p,\ R,\ g,\ \theta,$ and $I_{cm\ of\ pulley} = \frac{1}{2}m_pR^2$

c.) The system begins to accelerate. What is the magnitude of that *acceleration*? (Assume the acceleration is down the incline.)



m_1

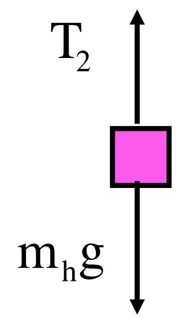


$$\sum F_{m_1,x}$$

$$T_1 - m_1 g\sin\theta = -m_1 a$$

$$\Rightarrow \quad T_1 = m_1 g\sin\theta - m_1 a$$

m_2

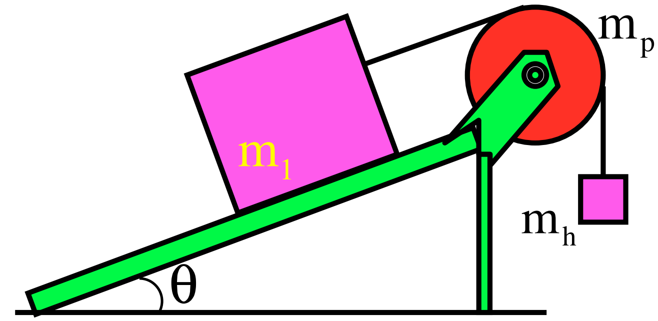


$$\sum F_{m_h,y}$$

$$T_2 - m_h g = m_h a$$

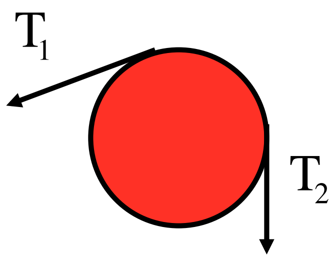
$$\Rightarrow \quad T_2 = m_h g + m_h a$$

$$m_1,\ m_h,\ m_p,\ R,\ g,\ \theta,\ \text{and}\ I_{cm\ of\ pulley} = \frac{1}{2}m_pR^2$$



c.) (con't.)

pulley



$$\sum \Gamma_{pulley}$$

$$-T_2R + T_1R = I_{pin}\alpha$$

$$\Rightarrow \quad -T_2\cancel{R} + T_1\cancel{R} = \left(\frac{1}{2}m_p\cancel{R^2}\right)\left(\frac{a}{\cancel{R}}\right)$$

$$\Rightarrow \quad -T_2 + T_1 = \frac{1}{2}m_pa$$

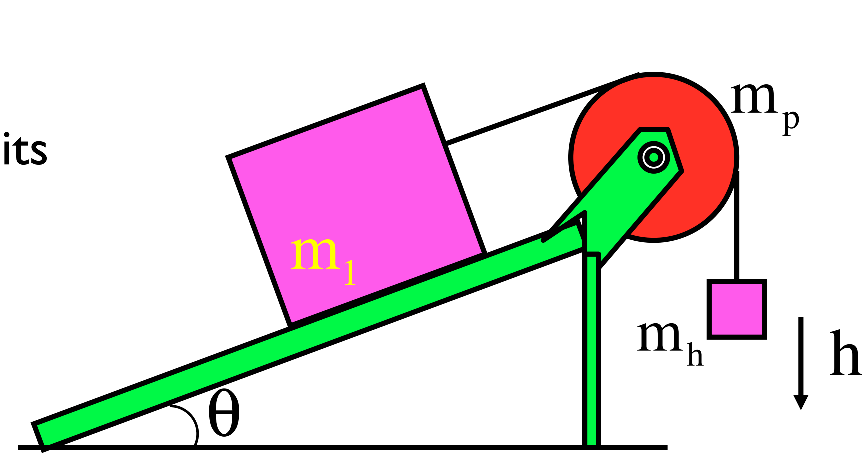
$$\Rightarrow \quad -(m_hg + m_ha) + (m_1g\sin\theta - m_1a) = \frac{1}{2}m_pa$$

$$\Rightarrow \quad a = \frac{-m_hg + m_1g\sin\theta}{\left(\frac{1}{2}m_p + m_1 + m_h\right)}$$

d.) What is the pulley's *angular acceleration?* $\alpha = \left(\dfrac{a}{R}\right)$ (good enough!)

$$m_1,\ m_h,\ m_p,\ R,\ g,\ \theta, \text{ and } I_{cm\ of\ pulley} = \frac{1}{2}m_pR^2$$

e.) The hanging mass drops a distance "h." What is its *velocity* magnitude at that point?



Note that if the hanging mass drops, the mass on the incline goes UP the incline:

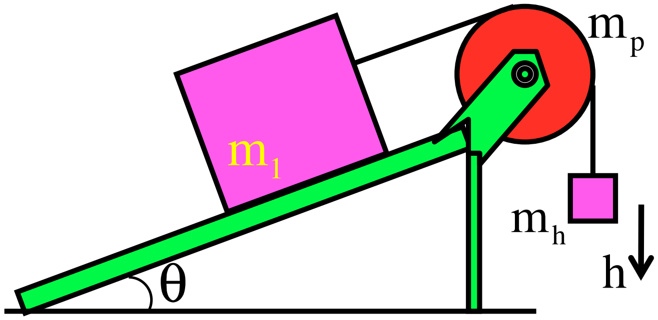
$$\sum KE_1 \quad \sum U_1 + \sum W_{ext} = \qquad \sum KE_2 \qquad + \quad \sum U_2$$

$$0 \quad + [m_hgh] + \quad 0 \quad = \left[\frac{1}{2}m_1v^2 + \frac{1}{2}m_hv^2 + \frac{1}{2}I_{pulley}\omega^2\right] + \left[m_1g(h\sin\theta)\right]$$

$$\Rightarrow \quad [m_hgh] = \left[\frac{1}{2}m_1v^2 + \frac{1}{2}m_hv^2 + \frac{1}{2}\left(\frac{1}{2}m_p\cancel{R^2}\right)\left(\frac{v}{\cancel{R}}\right)^2\right] + \left[m_1g(h\sin\theta)\right]$$

$$\Rightarrow \qquad v = \sqrt{\frac{2\left(m_hgh - m_1g(h\sin\theta)\right)}{m_1 + m_h + \frac{m_p}{2}}}$$

$$m_1, \; m_h, \; m_p, R, g, \theta, \text{ and } I_{\text{cm of pulley}} = \frac{1}{2} m_p R^2$$



f.) What is the *angular velocity* of the pulley at that point?

$$\omega = \frac{v}{R}$$

g.) What is the *angular momentum* of the pulley at that point?

$$L = I_{pin}\omega$$